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Leviathanian Fiscal Competition in Heterogeneous Country

Abstract

In this normative study of fiscal competition mechanism, we allow for various schemes of taxation, various mobility of tax-base, non-identical regions, and nonbenevolent governments. We examine the fundamental trade-off between “negative externalities” of the competition and benefits from its budget discipline. Some indicators of regional “rivalry” and “non-benevolence” are constructed for diagnosing marginal “over-taxing” in any region. It enables also “in-large” comparison of fiscal regimes. Under some restrictions on the country’s heterogeneity in tax rates, marginal “over-taxing” at competition signifies that switching to certain sort of tax co-ordination would deteriorate welfare.

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1 Introduction

In many economic areas, including international trade, oligopoly, and public economics, one compares outcomes of *independent* decisions of agents with possible results of their *co-ordinated* policies. Often co-ordination can improve efficiency, accounting for spillover effects (externalities) among decision makers. However, it may be harmful, when patterns of co-ordination are restricted, or social efficiency differs from goals of decision makers (e.g., in oligopoly). It is exactly the case in the field to be studied here - in fiscal federalism, at least when local governments are *not completely benevolent*, and unable to cooperate too sophisticatedly.

The dilemma “competition vs. co-ordination” appears here rather contradictory and this “constitutional choice” remains broadly debated in scientific and political communities. On the presence of tax competition in Russia and discussion around it one may see Kokovin and Kolomak (2000).¹ Analogous examples of tax competition within European Union, and the motives for its normative study, are brightly presented in Edwards and Keen (1996).² In this article (heavily relied on further, being referred to as E&K), the dilemma to be studied is formulated as follows:

“Is international tax competition - or, by the same token, tax competition between lower-level governments in a federal structure - a good thing or a bad thing? Or, to put the point more precisely (and in the form that it would be addressed):

Starting from the non-cooperative equilibrium, would the representative citizen benefit from, or be harmed by, some degree of international tax co-ordination?

Two widely divergent views [on this topic] dominate both the academic literature and the policy debate.” (E&K, p.114, emphasis added.)

One view can be called anti-federalist or “politically left”, because it believes in benevolent state, being concerned with raising the government’s power through more tax revenue (the theoretic development of this view originates from Oates [1972], it includes Zodrow and Mieszkowski [1986], Oates and Schwab [1988], and others). It supposes that several states may compete for mobile tax-payers by decreasing their local tax rates, like Bertranian oligopolists do compete by prices.³ Surely, the states can better achieve their fiscal goals if

¹The paper observes that inter-regional tax competition for capital in Russia in the form of regional tax relieves and tax holidays became quite widespread during 1990-s, until forbidding the relieves by law in 2001.

²One of the episodes described in Edwards and Keen (1996) is when Sweden reduced capital income tax to flat 30% in 1991, Norway followed the pattern with 28% in 1992, and Finland joined the race with 25% in 1993!

³Some authors believe this competition to become sufficiently fierce in globalization process: “...fiscal competition will wipe out redistributive taxes and reduce the tax system to one of mere benefit taxation.” (Sinn, 1994). By the way, this citation stresses an important thing: mobile tax-payers try to fly away from taxation *only* when it is to some extent *redistributive*.

“internalizing externalities” by ceasing competition and cooperating, i.e., becoming a cartel. It is exactly the aim of tax-co-ordination of regions (or just of centralized taxation) in this leftist view, since governmental goals are identified as social ones.

Another view can be called federalist, or “politically right”, since it is anxious with restricting inefficient redistributive activity of the states, who are supposed partially non-benevolent or completely “Leviathanian”. Starting from famous Tiebout paper (1956), following Brennan and Buchanan (1977), and many others, this strand of thought argued that fiscal competition based on tax-payer’s “voting by feet” may discipline non-benevolent local governments to spend tax revenue *only* in tax-payers’ interests (“benefit taxation”) and to refrain from unneeded (for the taxpayer) taxes.⁴ It was shown in literature (see survey by Wilson [1999] and more broad club/federalism theory in recent survey by S. Scotchmer [2002]) that the competition may yield Pareto-efficiency independently of governments’ non-benevolence. However, this requires some hardly-realistic conditions, including lump-sum scheme of taxation, absolutely free mobility of tax-payers, and, most important, *no need* in redistributive taxation (it is the rightist view on politics).

Edwards and Keen argue that “these two ... contrasting views clearly reflect profoundly different perceptions of policy-making... <and> ... of government”, so their mere observation bears “... no clear guidance ... for the pressing issues of practical policy-making. The central purpose of this (E&K) paper is to provide a framework within which these two sharply contrasting views of tax competition can be articulated and compared. For both, clearly, are extreme cases of a more general - and presumably more plausible - formulation in which policy-makers attach some value both to the welfare of their citizens and to the surplus that they are able to extract from the citizenry and put to their own uses.”⁵

A synthesis of these two divergent views seems necessary. To perform it we follow the approach suggested by E&K.⁶ They transform the dilemma ‘competition vs. co-ordination’ from the realm of beliefs into a *quantitative*, i.e., an empirical question. That means that **fiscal federalism may be good for one country, but bad for another, or good only for applying to a specific tax base**, depending upon the economic situation. In particular, E&K mainly speak of capital-income tax in Euro-Union, but in essence the authors develop a universal intuitively meaningful measurable economic index, or criterion to detect “local under-taxing”. Namely, “starting from the non-cooperative equilibrium, a small multilateral increase in the tax on mobile capital increases the welfare of the representative citizen if and only if *the elasticity of tax base exceeds the policy-maker’s marginal propensity to waste tax revenue*” (regions are supposed identical the tax is the same everywhere).

Indeed, our intuition about these countervailing effects expect that the greater is “rivalry”

⁴E.g., British government once resisted idea of European tax coordination, since with competition “the pressure on tax rates would be in general downwards, providing an essential antidote to the built-in pressures for increased public expenditure and taxation.” (cited from E&K and UK Treasury, 1988). We shall further interpret this upward pressure on taxation decisions as “populism”.

⁵In other words, from numerous papers we see, that contradictory views on competition efficiency arise, naturally, from different assumptions. Federalist or rightist view supposes: 1) public goods, beneficial only for taxpayers (i.e., benefit taxes), 2) rent-seeking government, 3) non-distortive (lump-sum) taxes. In contrast, unitarian or leftist view assumes the opposite at all three positions. So, some synthetic approach like E&K or the present one, is needed for moving forward in discussion.

⁶Besides, two other synthetic papers, combining countervailing effects into a unified framework are Keen and Marchand [1997], and Sorensen [2000]. The first approach is not appropriate for our goals since concentrated on specific type of budget spendings. The second present a quantitative general-equilibrium computer model with specific CES production functions and other particular hypotheses, also unsuitable for us.

of the competing regions, the more arguments exist for co-ordination, while the greater is “non-benevolence”, the more reasons we see for the opposite. In essence, E&K supports this view by logic, and suggests a particular appropriate *measures* of rivalry and non-benevolence.

The present setting extends E&K construction of empirical criterion of “good or bad” tax competition, attempting to make it more detailed and practical. Modifications include several aspects:

1) *Partial-equilibrium representation of ‘feasible economy’.*

Being concerned with corporate and capital-income taxes in Russian federation first of all, we believe that it is practical to resolve our constitutional dilemma *differently* for different economy sectors. Heterogeneity of economy in this dimension does matter. Say, it may turn out useful to leave taxes on small business completely in the regional discretion, while manufacturing of soft drinks, being very mobile across regions and very competitive, may bring more national welfare under tax co-ordination or federal taxation. Specific treatment is needed also for taxation of oil-and-gas industry with its huge natural rent and strong lobby (by the way, there was a serious struggle between federal and local authorities for the right to levy taxes on oil-and-gas).

These sector-separation ideas motivate our step aside from the long tradition (including E&K and everybody) of treating constitutional dilemma in general-equilibrium framework, to a new model. E&K describe the whole economy by the unique neoclassical production function, depending upon capital and labor inputs, and producing a single consumer good, which may become public good as well. The unique representative consumer-citizen of each region is at the same time the immobile employee, the mobile capital owner, and the immobile consumer of private and public goods altogether. Such modelling simplifies treatment, but shadows politically important considerations that we have in mind. Turning to *partial-equilibrium model* of some sector or industry, we shadow instead the inter-sector externalities and income-effect⁷ We believe that this inter-sector bias is less important, than the contradiction between the rich (citizens and regions) and the poor, the main contradiction to be addressed in resolving the dilemma.

In our model **the economy**, or the sector of economy to be taxed, has m regions, described by m different regional demand functions for some commodity, or by tax-revenue and welfare functions born by integrating these demands. (We speak of taxing *some* commodity at first, having in mind the most interesting particular commodity being capital.) These demands depend upon the nation-wide price of the commodity and on m local taxes (taxes can be simply added to the price like trade merges, or levied in more complex form). Thereby inter-regional tax competition is similar to local-resellers competition.

Therefore, the sector’s potential productivity in any region is measured by potential consumer surplus *and* taxes that it may generate. It is the demand curve’s integral minus (linear) costs. Such *monetary* measure of utility is traditional for partial equilibrium models, but needs some explanation here.

In our case, the most important and predominantly discussed application of the model is taxing *capital*. Here the regional demand for capital is the “investment curve”, i.e., the combination of all potential investment projects in the sector studied (see Section 1 and

⁷However, if applied to the whole economy, the particular version of our model turns out to be essentially identical to the traditional model, so the possible income-effect bias lies not in the partial-equilibrium model itself, but in its specific application.

Appendix for details). “Consumer surplus” takes the form of entrepreneur’s profit or residual, while the “pre-tax price” includes the interest rate paid by entrepreneur to capital owners. It equals “cost” which is the value of alternative use of capital by its owners, say, when applied abroad or in consumption. These owners (rentiers) are excluded from public welfare calculation for the reasons discussed in Appendix, only residual claimants represent the goals of business in this game.

2) *Non-trivial inter-regional mobility of the tax-base and variable taxation schemes.*

How harsh will be the tax competition? It depends upon the inter-regional capital mobility (fluidity). We incorporate the assumption on the degree of mobility (fluidity) right into the capital-demand function, making it dependent upon *all* taxes. Thus fluidity becomes a specific exogenous parameter of economy, distinct from economy’s productivity. It makes difference from the traditional approach. There ‘free’ *apriory* mobility of capital (free by legal right) means only equalization of after-tax profits, while the shape of the production function constitutes smaller or larger observed *aposteriory* capital mobility (fluidity across regions), i.e., some investment elasticity w.r.t. taxes. In contrast, here some sector or region may have the same productivity profile as another one, but quite different *aposteriory* mobility (fluidity) of capital.⁸ This ‘degree of substitutability’ of regions can not be modelled by the traditional production function, while our approach captures it.

Besides, the real capital-fluidity pattern depends upon the scheme of taxation: is it property tax, profit tax, VAT, or something else. Our model, unlike the traditional one, is appropriate to study and compare any of these schemes, applied to the chosen sector or economy.

3) *Heterogeneous population: conflict of interests between rich and poor citizens.*

In the traditional model, the unique “representative citizen” is the only player, who reflects social goals, opposing self-interest of the Leviathanian politicians. Instead we prefer to describe citizens as two *different* groups of population: businessmen and labor owners (typical households). It is much more realistic in Russia and in many other countries, so the conflict between these two groups of interests is at least as much important in the story about excessive taxation, as the conflict between the average citizen and Leviathan. Indeed, our topic is first of all the *redistributive* taxation. In Russia it means mainly taxes on business, when they are used for maintaining public goods for households. This process can be called “milking” of business taxpayer, in contrast with “beneficiary taxation” situation. Rightist political views (shared by the authors to some extent) look on such redistributive practice with suspicion, supposing its inefficiency. This practice can be a simple consequence of democratic process, dominated by majority of relatively poor households. They eagerly “milk” corporations, since underestimating the related bad feedback to wages and employment. May be, as a result not only the “average” citizen is worse off, but even the poorest part of population also loses, due to shrinking investment. I.e., political pressure towards “milking” is suspected here to be just a product of political *illusion* of the median voter, who do not realize the full consequences of her redistributive effort.

Nevertheless, it is not a scientific task to express *our* views on populism and corruption in political economy. What we should do (and do) in our setting, is just to assume, that the *federal legislative body* (constructing the federalist constitution) can have different degree of populism than populism of regional authorities, competing with each other. This gap in

⁸Say, manufacturing of drinks is more regionally-mobile business than services, though the two may have similar demand and cost curves.

motives between the two levels is supposed in this paper as much important, as the gap in their view on local corruption or unneeded spendings, which is the main focus of E&K (note, that even corrupted and populist federal legislator may be against corruption and populism at lower level!). As a result, the indicators of rivalry and non-benevolence should account for this two-fold gap, irrespective of its nature and sign. In particular, our analysis may help benevolent legislator to achieve her goals, but the same logic works for populist legislator as well.

4) *Heterogeneous country: conflict of interests between rich and poor regions.*

In contrast with tradition including E&K, we suppose a *heterogeneous country*, that means different, non-identical regions. It is not only because it is the case in Russia, which is our empirical field. The point is again in the conflict of interests, that may arise in any country from its heterogeneity and should be touched in the politico-economic discussion of federalism. Under heterogeneity, unlike the traditional homogeneous model, some regions may show positive indicator of need in additional taxation, while other show negative indicator. Theory must give an answer for this case: what is the socially-desired policy? It depends upon hypotheses on “plausible” policies. Can all regions agree on raising tax rate in one region and lowering tax rate in another at the same time? Can they agree on compensating the related losses of some regions by some transfers from others? We look pessimistically in both respects on the wisdom and sophistication of inter-regional political process, turning here to the next item of our story.

5) *Discrete change of political regime.*

The basic traditional question of “under-taxing criterion”, which justifies local (infinitesimally small) rise in tax rates, is only a step. We consider further a *global jump* from competition regime to co-ordination regime. Indeed, we hardly can believe that the co-ordination activity of regions, once started, can stop on infinitesimally small changes (similarly, competition can not stop when started). More realistic is to expect, that after the constitutional rules of the game start enabling cooperation, then the cooperation will evolve to full extent, up to some new equilibrium in this new co-ordination-game.⁹

But what kind of cooperation-equilibrium can occur? When there are more than 3-5 regions, we do not believe in the concept of the *core*, due to high negotiation costs. We assume instead some non-cooperative mechanism for reaching an agreement among regions. One of such mechanisms to be considered is *majority* or “*shifted-majority*” *voting* for unified tax rate (explained further), without any compensating transfers.¹⁰ It is the proxy for some more complex political mechanism with majoritarian properties, seeming realistic in Russia.

The findings of the present paper 1) adjust E&K local “under-taxing criterion” for our extended context, 2) use it to identify economic conditions, when *global* switch from competition to co-ordination is welfare justified.

Section 2 introduces our new model and discusses assumptions.

⁹It is similar to curing a market-failure with the help of governmental regulation: theoretically correct is to compare market-failure with the alternative government-failure, not with the ideal regulation or ‘small’ regulation. Exactly the same logic is used in the reverse situation: starting from coordinated taxes, is it socially beneficial to legalize *full-extent* tax competition? “Small” (semi-)competition is hardly understandable.

¹⁰Though massive inter-budgetary transfers and political struggle for them exist in Russia, we take them fixed in the game considered, to separate these complex issues.

In Section 3 the question of welfare comparison between competition and co-ordination “in large” is decomposed into two sub-questions. 1) Are taxes globally lower in competition than in co-ordination in certain sense? 2) Are they at the same time still “locally too high”, in the sense that small tax reduction increases welfare? If both answers are positive, then there is no need in “jumping” from competition to co-ordination. (This logic can be reversed to estimate a shift from co-ordination to competition.)

Section 4 deals with the second question - local-shift analysis, to extend E&K criterion of social gains from small additional taxation. Related economic indicators of regional ‘rivalry’ and ‘non-benevolence’ are developed to account for populism and heterogeneity, as well as for corruption. Namely, if we observe small rivalry in tax-competition, jointly with big populism and/or big corruption - then it is the signal that small tax stimulation is not socially beneficial. If combined with tax-increase prognosis from Section 5.2, such diagnosis definitely advises us *not* to undertake ‘big bang’ from competition to co-ordination: let Leviathans compete rather than collude.¹¹

Section 5 deals with the tax increase after switch to co-ordination (defined and discussed in this section): will average tax rate really rise? The positive answer seems a common wisdom, but it is really justified only under additional assumptions like symmetric and almost-symmetric (in certain sense) equilibria. Alternative assumption is ‘big’ mobility (we show what it means), which bores famous ‘race to the bottom’. Some assumptions of this kind are needed to escape ambiguity in reaction of heterogeneous economy to constitutional changes.

Conclusion summarizes. Appendix contains some additional explanations, figures, and examples.

2 New Model of Redistributive Tax Competition

As we have explained, our specific object of interest is “Leviathanian milking of business”. (“Milking” means re-distributive taxation, like in Oatsian tradition: the state is taxing *business* to finance public goods for *households*, unlike Tieboutian tradition.) Trying to formulate a “minimal” abstract model of this game, and still capture all important effects, we came to the following partial-equilibrium setting. It modifies the traditional setting from Zodrow & Miezhevskii (1986) and, more closely, from E&K – by assuming no income effects (supposing quasilinear utilities). Then it is convenient to replace production function and preferences by demand functions, or by “tax base function” and “residual-welfare function” in the same role. As we have explained, in this case tax base *mobility* (aposteriori fluidity) appears to be a feature separate from economy’s productivity, that allows studying different forms of taxation for the same country. Besides, we drop unneeded specific assumptions on regional homogeneity etc. Our attempt is to make the model *natural* for the theory of redistributive tax competition, and *useful* practically.

¹¹Note that such arguments in favor of competition could be enforced if we considered also benefits from public goods to business and related Tieboutian forces. Thus our simplified treatment is sufficient to support competition, but insufficient to reject it.

2.1 Entrepreneur's residual and tax revenue: what is the pie to be splitted?

There are m regions or markets $i = 1, \dots, m$ each ruled by some authority named here “governor”. Each governor i chooses a triple $(t_i, g_i, l_i) \in R_+^3$ subject to some constraints, where $t_i \geq 0$ denotes *tax rate*, $g_i \geq 0$ is public expenditures for maintaining a *public good*, $l_i \geq 0$ is governor's own *luxury spending*, or other unnecessary for the region, wasteful governmental spending.

There is some activity, or commodity, or production factor traded in the country, and optimizing taxation of this trade is our question. For definite interpretation, further this factor is called **capital** (it is the most important factor for regional development, but the model can be applied also to something else). A region i is characterized by two functions, related to this good and its taxation.

Tax-revenue function $T_i = T_i(t_1, \dots, t_m)$ denotes the amount of taxes that can be collected in region i . Note that it may be dependent upon all taxes of competing regions, and presumably it should have maximum (Lafferian point) w.r.t. region's own tax t_i . In particular, the function may be constructed from some *tax-base function* $K_i = K_i(t_1, \dots, t_m)$ so that $T_i(t_1, \dots, t_m) = t_i K_i(t_1, \dots, t_m)$. It is mentioned just to note, that “tax base” is similar to demand functions of monopolistic-competition model, taxes are like prices, and tax revenue is like profit. A governor acts like a local monopolist on the demand side, at the same time competing with other regions for supply (of capital).

Tax-payers' residual-wealth function $V_i = V_i(t_1, \dots, t_m)$ plays the role of ‘consumer surplus’; it denotes net monetary welfare of tax-payers (of corporations or entrepreneurs) in region i , i.e., the residual remaining after paying interest (cost of capital, assumed constant) and taxes. It does not include benefits from public goods maintained on these taxes. (It should not be mixed with tax-payers' “full” welfare or indirect-valuation function $V_i^{full}(t) = V_i(t) + T_i(t)$, which is similar to the integral of demand for capital, i.e., total possible net of interest *gross* profit, while V_i is *net* after-tax entrepreneur's residual.) In particular cases (not always) both these elements of the model: tax revenue and residual wealth - can be derived from *one* basic element like “tax base” or demand function, as explained in Appendix. So, tax competition reminds “monopolistic competition” among “intermediaries” (re-sellers). One of the departures is the following.

2.2 Legislator's and governor's objectives: what is (non-) benevolence?

What can be the local authority's motives? We shall assume that the **governor's goals** do include, in one or another way, three different objectives: 1) welfare of *business* (with a weight denoted $\beta \geq 0$), 2) utility of households from public good (with weight $\gamma \geq 0$), and 3) governor's own “luxury” (with weight $\lambda \geq 0$). For simplicity of exposition, we combine them in separable objective function

$$U_i^{\beta\gamma\lambda}(t, g_i, l_i) = \beta V_i(t) + \gamma u_i(g_i) + \lambda w_i(l_i),$$

where, unless $\beta_i = 0$, the weights may be normalized conveniently for our interpretation, as $\beta_i = 1$, so that all benefits be in dollars or roubles (so, further we often drop symbol β .) An increasing function $u_i = u_i(g_i)$ (often further supposed the same for all regions) denotes region's utility from public spending. It is measured in monetary terms compatible

with business gains $V_i(t)$. It includes mainly households' satisfaction, but some benefits for business may be assumed also, unless it causes non-convexity of optimization programs. "Satisfaction from waste" function $w_i = w_i(l_i)$ similarly denotes governor's satisfaction from luxury spendings l_i . Budget constraint connects revenue with these spendings. More specifically, the region's income, splitted between the two goals, may include some fixed money transfer (subsidy) $s_i \in R$ from the federal sources ($s_i < 0$ means transfer *from* the region). So, region's budget constraint is

$$l_i = T_i(t) + s_i - g_i.$$

Further, an important element of our reasoning is **social welfare function**. It must reflect *social planner's*, i.e., *federal legislator's* objectives. We mean the agent who makes the named constitutional choice in federalism: co-ordination or competition. She can differ in 'benevolence' from regions. First of all, she may be unsatisfied with regional luxury. As to her balance between business and households' needs, we do not discuss, whether this balance is close to ours, or to some 'theoretical'. The question does not weight much, since *we* are not the policy makers. Instead we just take legislator's preference for a standard, to develop advices *for her*.

So, crucial for the sequel is only the difference between governor's goals and federal legislator's goals $(\beta_*, \gamma_*, \lambda_*)$. For the legislator, it is natural to assume all weights of regional business $\beta_* = \beta_{i*} = \beta_{j*} = 1$, treating all local businessmen equally. Similarly, we can assume, that all consumers of public goods are treated equally by the legislator, and their utility $u_i(g_i)$ from different levels g_i of public good is normalized in terms of willingness-to pay for it, so that social optimum must be achieved at point $\dot{u}_i(g_i) = 1$ (dollar of marginal benefits equals dollar of costs). Then benevolent weights can be supposed $\gamma_{i*} = 1 = \beta_*$. As to the legislator's attitude to luxury, we shall suppose her (not quite realistically) opposing luxury on regional level completely: $\lambda_* = \lambda_{i*} = \lambda_{j*} = 0$. Therefore we formulate the "benevolent" whole-country's objective function simply as

$$U_{all}^{ben}(t, g, l) = \sum_i [V_i(t) + u_i(g_i)].$$

Using this objective function, we do not seek for first-best social optimum, but compare different non-optimal situations with each other in "social efficiency". Mainly we compare equilibrium and co-ordination solutions to each other. We mention also the maximum of this "benevolent" function w.r.t. centralized taxes only (t_1, \dots, t_m) (spendings g, l being optimized by regions afterwards). It may be called the "second-best" social optimum, while "third-best" is similar maximum with additional constraint of uniform taxation: $t_i = t_j \forall i, j$. But both these optima do not play important role in our discussion, since federal legislator hardly have evident way to implement such policy, due to agency-problem and lack of information about actual tax base. Instead we should compare *practically admissible* solutions like equilibrium or co-ordination.

Let us further discuss **the difference between local and federal goals**, to motivate our hypotheses.

'Benevolence' of local governments would mean, that regional administrations not only hate luxury, but also choose the same balance as the central authority ($\beta/\gamma = 1$) in trade-off between taxpayers' (businessmen, mainly wealthy people) and public good consumers

(households, mainly poor people). The latter may be not the case for at least two reasons to be explained: 1) populism; 2) tax-exporting.

We argue that even a luxury-hating ‘honest’ government may behave as non-benelolent “Leviathan” in respect of over-taxing, because of its specific position in the political game. The name of “populism” we give to all situations, where local governments are pursuing goals of their voters (mostly households) too-straightforwardly. Average household voter (at least in Russia) is likely to *underestimate* negative consequences for him (in the long-run) of too-high taxes on business. He sees only direct short-run benefits from ‘milking’ corporates. (Who will vote against taxes on somebody else?) This tendency is further enforced by tax-exporting motive: when sufficient part of capital owners are outsiders, then a region is interested in maintaining own public goods on *their* expenses.¹² Of course, populism could be outweighed by lobbying activity of *big* business. But this pressure is hardly sufficient to reduce taxes on *average* entrepreneur. Rather lobby seeks personal exemptions. This makes us to believe in *excessive* populism of regions relatively to federal legislator in most countries, and to discuss further this hypothesis more than the opposite.

2.3 Solution concept and assumptions: what is tax-competition equilibrium?

Now we formulate the concept of the game among regions.

We assume that a governor maximizes the objective function $U_i^{\beta\gamma\lambda}(t, g_i, l_i)$ subject to the described budget constraint and to natural constraints on variables $g_i \geq 0, l_i \geq 0, t_i \in [0, 1]$ (here interval $[0, 1]$ should be understood as *normalized* one, substituting some real legal or natural bounds $[\underline{t}, \bar{t}]$ on taxes).

Definition 1 Subsidy values s_i given, (*Oatsian*) *tax-competition equilibrium* is a Nash equilibrium $(\bar{t}, \bar{g}, \bar{l}) \in R_+^{3m}$ in the game of regions, each region solving the program:¹³

$$\begin{aligned} U_i^{\beta\gamma\lambda}((t_i, \bar{t}_{-i}), g_i, l_i) &\rightarrow \max_{(t_i, g_i, l_i)}, \text{ s.t. } g_i \geq 0, l_i \geq 0, t_i \in [0, 1], \\ T_i(t_i, \bar{t}_{-i}) + s_i - l_i - g_i &= 0. \end{aligned}$$

Such “competition regime” should be compared with some kind of regional “co-ordination”, introduced below in special section.

To analyze competition equilibria we shall use throughout the following hypotheses, ensuring solutions’ existence and their needed properties.

Assumption 1 All functions V, T, u, w are twice continuously differentiable, positive-valued,¹⁴ functions u, w are increasing and strictly concave, u satisfies Inada’s condition

¹²We suppose, that populism pressure on federal legislator is weaker than on regional one. At least, on federal level such complex question as constitutional choice has better chance to escape this pressure, than question of tax rates, obviously being a money-splitting one. Another source of populism – tax-exporting – is also weaker on federal level, concerning only foreigners.

¹³Comparing this definition with its specific realization in the above example of “distorting taxes”, one can note that governors here are supposed wise enough to predict influence of their taxes on capital-equilibrium price $p^*(t)$. It is not too realistic, but we suppose it making a little difference from “myopic behavior” assumption in our context. Indeed, interest rate $p^*(t)$ is hardly sensitive enough to a single tax, to change the main results.

¹⁴This matters for comparative statics w.r.t. γ, λ .

($\dot{u}(0) = \infty, \dot{u}(\infty) = 0$), $V_i(t_1, \dots, t_m)$ decreases w.r.t. t_i , increases w.r.t. $t_{j \neq i}$, being strictly concave w.r.t. t . Function $T_i(t_1, \dots, t_m)$ is concave w.r.t. t_i , increasing w.r.t. $t_{j \neq i}$ and productive, i.e., for all $t_{j \neq i}$ there exists $t_i : T_i(t) > 0$. Besides, we shall always suppose transfers s_i strongly feasible in the sense: $T_i(t) + s_i > 0 \forall t \in [0, 1]^m$.

Let us discuss the assumptions. The T positivity assumption, admitted for all taxes $t \in [0, 1]^m$ ensure non-emptiness of interiority of admissible sets (regularity condition) for all individual and collective optimization problems discussed further. It is a mild assumption when interval $[0, 1]$ is understood as *normalized* one, or/and *this* tax t is not the only source of regional income. Together with functions (strict) concavity it ensures applicability of Kuhn-Tacker theorem to get F.O.C. (first-order conditions), and uniqueness of individual solutions, and applicability of Nash theorem to ensure existence of our equilibria (standardly).

The concavity assumption adopted is not quite innocent. In particular, when $T_i(t) = t_i D_i(t)$, $V_i(t) = p_0 D_i(t) + \int_0^{p_0+t_i} D_i(\tau)$, it restricts the shape of demand function D_i . Still, linear demands and many other demand functions, even non-concave, satisfy it.

Assumption 2 . The competition equilibrium solution t^* lies below all Lafferian points for all regions, i.e., $\frac{\partial}{\partial t_i} T_i(t^*) > 0 \forall i$.

This natural assumption on technology and preferences restricts also the solutions studied. It seems to follow from other assumptions, but clarifying it seems unimportant.

Assumption 3 . Welfare function $U_{all}^*(\cdot)$ is concave.

In appendix we argue that under A1, the sufficient (far from necessary) condition yielding A3 is the specific restriction on relative concavity of public-goods and luxury satisfaction functions u, w . When one of them is “**more quickly satiable**” in certain sense, then the demand for this commodity is concave. We say that for an agent with utility function $U(g, l) = u(g) + w(l)$, his demand for g is “more quickly satiable” than demand for l when his response-function $g^*(T)$ to income T (under budget constraint) is concave. It is hard to tell, is it a realistic assumption or not, to suppose governor’s demand for public goods being “more quickly satiable” than demand for luxury, as we need. If both functions are homogeneous of the same order, this holds.

On the other hand, concavity of welfare function can follow from other sources, even when $g^*(\cdot)$ is not concave. The reason for assumption is that it is tightly connected with uniqueness of equilibria and it is very hard to make welfare predictions ‘in-large’ without it.

Assumption 4 . Cross-derivatives of tax-revenue and residual functions have signs $\ddot{T}_{iik}^* < 0, \ddot{V}_{iik} < 0$.

This assumption is used only in one section for comparing size of tax rates of coordination and equilibrium and it is explained when used.

3 Logic of comparisons “in-small” and “in-large”: separation formula

Generally, constitutional choice in fiscal area arise from agency problem, that forces some discretion power to be delegated downwards (in particular, we suppose regional productive possibilities as well as ‘budget hunger’, populism and luxury unobservable from outside). Unfortunately, we can not touch this choice too broadly here, due to our inability to directly model the informational structure of the real game between federal and local govern-

ments, and informational costs preventing certain forms of centralization (including Pigouvian taxation or lump-sum taxation). Instead, we leave our assumptions about plausible and implausible political mechanisms out of the model, and compare only several *particular constitutional solutions*, often discussed or practiced: tax-competition equilibria and some co-ordinated equilibria, defined later. Now we should speak of the whole class of various co-ordinated equilibria, to convince that all can be compared somehow to competition.

Should we discuss co-ordination ‘in-small’ or ‘in-large’? We have mentioned that ‘infinitesimally-small’ co-ordination of taxes (studied in E&K) is theoretically interesting, but hardly realistic. More probable is that regions, after acquiring rights or ability to co-ordinate taxes, will not stop on small changes. They can switch to completely new co-ordinated equilibrium. So, constitutional choice should be looked upon as a *global* ‘switch’ from competition to some other distinct regime, when comparing welfare.

Generally, under our assumptions such welfare comparison is ambiguous, since countervailing forces are involved (even the common-wisdom downward pressure of competition upon taxes is sometimes questionable in heterogeneous country, as we show later on). However, there are certain cases with clear answers. Our goal now is to distinguish these cases and to develop their indicators, answering the question:

What economic variables should we measure to decide, that switching from a competition-equilibrium to some co-ordinated equilibrium (or vice versa) is socially beneficial?

To answer the question, there can be a general approach (applicable to many types of co-ordination regimes) that combines reasoning ‘in-small’ with global comparisons, as we shall see now.

Indirect welfare function and “separation”

To compare welfare at any two states of economy, we can use marginal information at one of them and linearize the estimated welfare function, exploiting concavity like in analysis of monopoly (see Varian, 1985).¹⁵ To accomplish this program, let us express economy’s true welfare function $U_{all}^{ben}(t, g, l) = \sum_i [V_i(t) + u_i(g_i)]$ in terms of taxes only. It means taking into account that region’s public expenses g_i are optimized dependent on budget constraint $g_i \leq T_i - l_i + s_i$ together with luxury spending l_i . Moreover, $g_i = g_i^*(T_i, \gamma_i, \lambda_i)$ must be the solution to region’s spending problem under special assumption on budget revenue: $T_i = T_i^*(t_1, \dots, t_m)$. Then we can define an *indirect welfare function*

$$U_{all}^*(t) = \sum_i [V_i(t) + u_i(g_i^*(T_i^*(t), \gamma, \lambda))].$$

This function takes into account impossibility to overcome regional non-benevolence of spendings: choice between public goods and luxury is supposed out of central control. So, its maximum can be called a *second-best social optimum* w.r.t. tax profile $t = (t_1, \dots, t_m)$. (This second-best is expressed by the best point \hat{w} on the indifference-curves map of indirect welfare function U_{all}^* on Fig. 1.)

It can happen that competition-equilibrium coincides with the second-best optimum \hat{w} . (Indeed, to build such an example, one can take any economy with benevolent identical regions and start increasing all populism parameters $\gamma_i = \gamma_j$ from 1 to ∞ simultaneously until optimality is achieved: $g_i^* = T_i^*(t) = t_{i\gamma}^* D_i(t)$. Under our assumptions, the

¹⁵As we have mentioned, it is not easy to make any predictions without some serious restriction like concavity. Even with concavity, a lot of ambiguity remains.

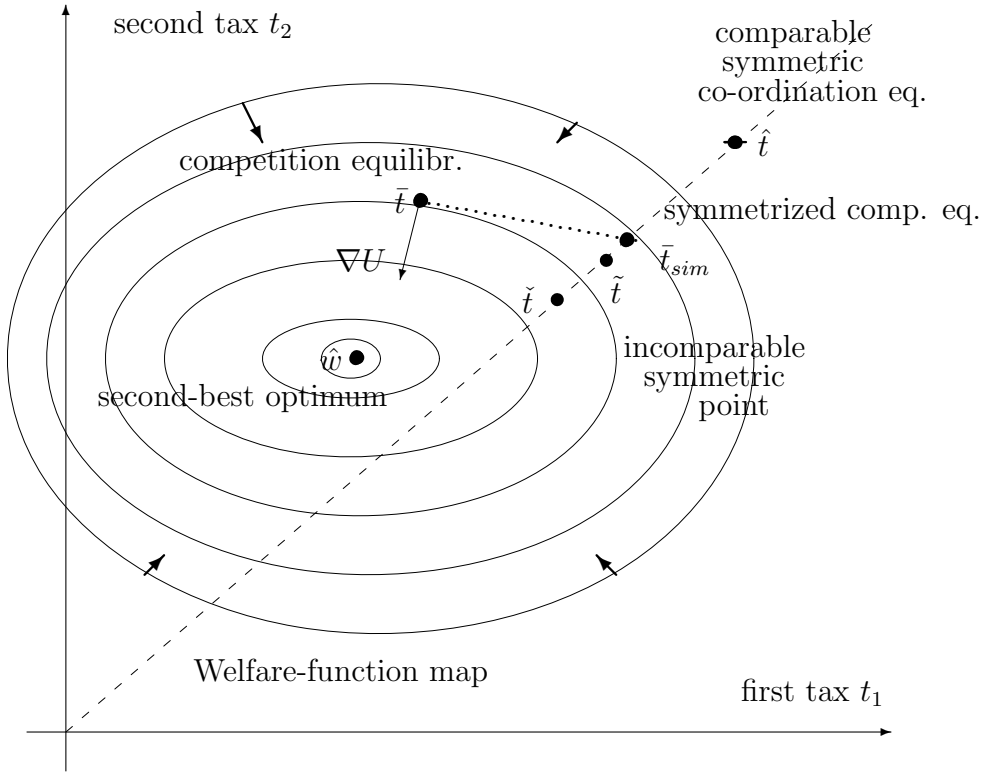


Figure 1: Welfare function $U_{all}^*(t_1, t_2)$, competition and co-ordination points in the case of over-taxing.

related comparative statics of the tax rate can be proved continuous, starting lower than optimum, and finishing higher than it.) It is possible, since on one hand, competition ignores externalities, but on the other - it hampers Leviathanian motives. These forces *can* exactly outweigh each other in specific cases.

More generally, when competition is better than co-ordination? In terms of Fig. 1 the question means: is the competition point \bar{t} situated *higher* than co-ordination point \hat{t} on the hill, described by the map of welfare function $U_{all}^*(t_1, t_2)$?

To compare any tax regime \bar{t} with an alternative regime \hat{t} , we can standardly use concavity of our welfare function (concavity is **Assumption 3** and our picture obeys it). By linearizing $U_{all}^*(\cdot)$ at \bar{t} (as described by dotted line), we build a loose estimate of welfare gains from switching to alternative point \hat{t} in terms of derivatives (separation formula):

$$U_{all}^*(\hat{t}) - U_{all}^*(\bar{t}) \leq (\hat{t} - \bar{t}) \nabla U_{all}^*(\bar{t}) = (\hat{\tau} - \bar{\tau}) \sum_{i=1}^m \frac{\partial}{\partial t_i} U_{all}^*(\bar{t}) . \quad (1)$$

Here scalars $\hat{\tau} := \tau_U(\hat{t})$, $\bar{\tau} := \tau_U(\bar{t})$ are the ‘*weighed average tax rates*’ at both regimes: $\tau_U(t) := \sum_{i=1}^m \xi_i t_i$, where weights $\xi_i = \frac{\partial}{\partial t_i} U_{all}^*(\bar{t}) / \sum_i \frac{\partial}{\partial t_i} U_{all}^*(\bar{t})$ may be interpreted as individual normalized “under-taxing indicators” (they can have positive or negative sign). The scalar $\sum_i \frac{\partial}{\partial t_i} U_{all}^*(\bar{t})$ is “aggregate under-taxing indicator” at point \bar{t} . Negativity of the right-hand side here (in particular occurring when each component $(\hat{t}_i - \bar{t}_i) \frac{\partial}{\partial t_i} U_{all,i}^*(\bar{t})$ is negative) is sufficient for old regime (competition \bar{t}) to be better than the new one. Therefore:

When the economy is overtaxed at initial regime \bar{t} in the sense of separation inequality

$$(\hat{t} - \bar{t}) \nabla U_{all}^*(\bar{t}) = (\hat{\tau} - \bar{\tau}) \sum_{i=1}^m \frac{\partial}{\partial t_i} U_{all}^*(\bar{t}) < 0,$$

comparatively to an alternative regime \hat{t} , then switching to the alternative regime would decrease social welfare.

¹⁶

In essence, this formula enables to transfer ‘in-small’ (marginal) indicator of over-taxing into ‘in-large’ (global) indicator. Moreover, unlike Edwards and Keen’s ‘in-small’ indicator, arranged only for symmetrically-identical regions and symmetrically rising taxes, this formula qualifies the whole *set of directions* of changes that bring decrease in welfare, for rather general situations (the set is half-space above the dotted line).

Geometrically, in example on Fig. 1 black point $\bar{t} = (\bar{t}_1, \bar{t}_2)$ denotes taxes at competition equilibrium, while black point $\hat{t} = (\hat{\tau}, \hat{\tau})$ denotes specific *uniform* co-ordination, that lies on the bisectrix. Point $\bar{t}_{sim} = (\bar{\tau}, \bar{\tau})$ denotes the ξ -symmetrized equilibrium \bar{t} , i.e., the ξ -projection of \bar{t} onto the bisectrix, that uses normalized gradient ∇U components ξ_i as weights (non-orthogonal projection). The (infinite) dotted projection-line $\bar{t} \bar{t}_{sim}$, orthogonal to gradient ∇U , divides the plane into two half-planes. Our formula says that the whole (right-upper) half-plane opposite to gradient ∇U is worse than \bar{t} . It is only a sufficient, not necessary condition: when co-ordination point is situated like point \hat{t} , then it is incomparable

¹⁶This method of comparison is reversible, but with care. When seing average- *undertaxing* in some regime and supposing average tax to *decrease* after turning to another one, the legislator also should not switch regimes. But nothing can be said for the opposite direction of tax change: concavity do not allow *two-side* estimates.

to \bar{t} within “separation inequality”, though being worse than \bar{t} . On the other hand, the picture shows, that simple non-weighted averaging of taxes could mislead our intuition. Namely, competition \bar{t} can have simple-average tax rate lower than some new point \check{t} (i.e., $\sum_i \bar{t}_i < \sum_i \check{t}_i$), and “average over-taxing” diagnosis fulfilled ($\sum_i \frac{\partial}{\partial t_i} U_{all}^*(\bar{t}) < 0$), but still bring less welfare than point \check{t} .

Thus with the help of the above assertion and its illustration we have divided the primary question (when competition is better?) into two questions.

- 1) Are taxes in average (in weighed average) lower at competition than at co-ordination?
- 2) Are they at the same time still excessively high marginally (‘in small’), in the sense that small tax reduction at competition increases welfare?

The below sections approach these two questions, starting from the second one.

Methodologically, note that in such comparison we are using only current *local* information to predict consequences of *global* shifts in the tax space (t_1, \dots, t_m) . It seems to be *the only possible* method. Global switching of regimes necessarily occurs blind, because it is hardly possible to get reasonable information about quite different state of economy where we have never been. Would-be elasticities and volumes of trade are unknown. That’s why we build comparisons only on current information (on initial state).

4 Welfare comparison ‘in-small’: local criterion of under-taxing

Let us focus on the assertion, which is a common wisdom in Oatsian tradition. It says that non-regulated competition equilibrium is likely to have “too-low” taxes (lower than welfare requires) since it ignores positive externalities among regions. Is it true under non-benevolent governments?

Externality effect may be out-weighed by luxury-loving or by “populism”. So, generally, “under-taxing” is as much possible at equilibrium, as “over-taxing”. How can we decide, which tendency is stronger in current real situation? It is a **quantitative empirical question**, to be discussed in this section. We attempt here to modify the method of measuring under-taxing, suggested by E&K, for more broad environment, including non-identical regions and populism.

Formally, measuring “under-taxing” means to estimate the gradient of welfare function U_{all}^* , or some its aggregate.¹⁷

To express marginal welfare increase in convenient terms, we study a very simple direction of tax change: an ort $\Delta t = (0, \dots, \Delta t_i, \dots, 0) > 0^m$, i.e., i -th component of the gradient ∇U_{all}^* . Economically it means to investigate the impact of some “small” tax stimulation at competition equilibria, that achieves increase of *only one* tax.¹⁸ It means to study only the “first reaction” to the increase in one tax, i.e., only ‘direct effects’ of the increase, including flight of capital from the region taxed, changes of tax revenues and spendings everywhere, but not changing taxes further towards a new equilibrium. So, only public goods

¹⁷In particular, E&K study special aggregate: impact of symmetric ($\Delta t_i = \Delta t_j$) increase of taxes on U_{all}^* , that is reasonable in their similar-regions setting.

¹⁸We should focus now only on the welfare consequences of tax rise, not on *how* Δt_i is stimulated. For instance, we can imagine an order from the federal body to rise one regional tax t_i and to keep other taxes unchanged.

and luxury spending $(g^*(T(t)), l^*(T(t)))$ will be adjusted to the new tax. Then, assuming interior solutions $(l, g > 0)$, the relevant increase in social welfare can be calculated from the full differential of our function for tax shift Δ_{t_i} :

$$\begin{aligned}\Delta U_{all}^*(t) &= \Delta_{t_i} * \frac{\partial}{\partial t_i} (\sum_j [V_j(t) + u_j(g_j^*(T_j(t)))])|_{t=\bar{t}} = \\ &= \Delta_{t_i} (\sum_j [\dot{V}_{ji}(\bar{t}) + \dot{u}_j(\bar{g}_j) \dot{g}_{jT}^*(\bar{T}) \dot{T}_{ji}(\bar{t})]).\end{aligned}$$

Here magnitude $\dot{g}_{iT}^*(T)$ is interpreted as *the share of increase in regional budget, going to expenditures on public good*.

(This $\dot{g}_{iT}^*(T)$ can be expressed from spending-optimization problem as a solution's characteristic $\dot{g}_{iT}^*(T) := \frac{\lambda_i \ddot{w}_{ij}(l_i)}{\lambda_i \ddot{w}_{ij}(l_i) + \gamma_i \ddot{u}_{ii}(g_i)}$. Under our conditions, it satisfies relation $0 \leq \dot{g}_{iT}^*(T) \leq 1$.¹⁹ Besides $\dot{g}_{iT}^*(T, \gamma, \lambda) = 0$ when $\gamma = 0$ and $\dot{g}_{iT}^*(T, \gamma, \lambda) = 1$ when $\lambda/\gamma = 0$, (though in the case of discontinuity of $\dot{g}_{iT}^*(T, \gamma, \lambda)$ this does not imply $\dot{g}_{iT}^*(T, \gamma, \lambda) \rightarrow 1$ when $\lambda/\gamma \rightarrow 0$). Under homogeneous objective function $(\gamma u(g) + \lambda w(l))$ or under weaker similar conditions, function \dot{g}_{iT}^* monotonically decreases from 1 to 0 w.r.t. λ , and increases w.r.t. γ .)

Now take the competition-equilibrium's first-order conditions:

$$\dot{V}_{ii}(\bar{t}) = -\gamma_i \dot{u}_i(\bar{g}_i) \dot{T}_{ii}(\bar{t}), \quad T_i(\bar{t}) + s_i - \bar{g}_i - l_i = 0,$$

(we suppose here $l > 0$, while another version $(\lambda = 0, l = 0)$ is obtained as particular cases of this one). Substituting them into $\Delta U_{all}^*(t)$ we get

$$\begin{aligned}\Delta U_{all}^* &= \Delta_{t_i} (\sum_{j \neq i} [\dot{V}_{ji}(\bar{t}) + \dot{u}_j(\bar{g}_j) \dot{g}_{jT}^*(\bar{T}) \dot{T}_{ji}(\bar{t})] + \\ &\quad + (\dot{g}_{iT}^*(T) - \gamma_i) \dot{u}_i(\bar{g}_i) \dot{T}_{ii}(\bar{t})).\end{aligned}$$

Similar is formula for gradient

$$\nabla U_{all}^* = \sum_{j \neq i} [\dot{V}_{ji}(\bar{t}) + \dot{u}_j(\bar{g}_j) \dot{g}_{jT}^*(\bar{T}) \dot{T}_{ji}(\bar{t})] + (\dot{g}_{iT}^*(T) - \gamma_i) \dot{u}_i(\bar{g}_i) \dot{T}_{ii}(\bar{t}).$$

Its i -th component positivity is equivalent to

$$\frac{\sum_{j \neq i} [\dot{V}_{ji}(\bar{t}) + \dot{u}_j(\bar{g}_j) \dot{g}_{jT}^*(\bar{T}) \dot{T}_{ji}(\bar{t})]}{\dot{T}_{ii}(\bar{t})} > (\gamma_i - \dot{g}_{iT}^*(T)) \dot{u}_i(\bar{g}_i).$$

If this relation is true, then some small tax-increasing stimulation is needed. This “*under-taxing criterion relation*” that we discuss further can be reformulated in terms of new notations L (“Leviathanian index” or “leftist index”) and R (“rivalry index”):

$$L_i(\gamma, \lambda) := 1 - \frac{\dot{g}_{iT}^*(\bar{T})}{\gamma_i} < \frac{\frac{\partial}{\partial t_i} U_{-i}^*(\bar{t})}{-\frac{\partial}{\partial t_i} V_i(\bar{t})} =: R_i(\gamma, \lambda), \quad (2)$$

¹⁹Indeed, $\{\gamma_j u_j(g_j) + \lambda_j w_j(l_j) \rightarrow \max \text{ s.t. } g_j + l_j \leq T_j\}$ is, in essence, the consumer's problem. Property $\dot{g}_{jT}^*(T) > 0$ is strict normality (of demand for g), and $0 \leq \dot{g}_{jT}^*(T) \leq 1$ says that l is not an inferior good, moreover, an additional unit of income is spent on both commodities, when $0 < \dot{g}_{jT}^*(T) < 1$. Both non-strict inequalities are guaranteed (strictness $0 < \dot{g}_{jT}^*(T)$ - with Inada's condition on u) for our separable objective function $\gamma_j u_j(g_j) + \lambda_j w_j(l_j)$.

where $\frac{\partial}{\partial t_i} U_{-i}^*(\bar{t}) = \sum_{j \neq i} [\dot{V}_{ji}(\bar{t}) + \dot{u}_j(\bar{g}_j) \dot{g}_{jT}^* \dot{T}_{ji}(\bar{t})]$. In particular, $\dot{g}_{iT}^*(\bar{T}) = 1$ in the case of $\lambda = 0$ (no luxury-loving).

Thus we have obtained the assertion, which establishes local *under-* or *over-taxing diagnosis* for a region:

Starting from (Oatsian) tax-competition equilibrium, a small increase of i -th tax (other taxes unchanged) rises social welfare if and only if inter-regional rivalry exceeds non-benevolence: $L_i(\gamma, \lambda) < R_i(\gamma, \lambda)$, and it deteriorates welfare in the opposite case: $L_i(\gamma, \lambda) > R_i(\gamma, \lambda)$.

What remains for local (over-) under-taxing is to add some rough considerations about *measuring* rivalry and Leviathan indicators in competition situation.

4.1 Can we measure rivalry and non-benevolence?

Let us interpret the obtained ‘under-taxing criterion’ and compare it with E&K’s analogue, reformulating it in measurable (?) terms.

1) **The left-hand side** of “under-taxing criterion”: $L_i = 1 - \frac{\dot{g}_{iT}^*}{\gamma_i}$ is the “Leviathanian index” or “corruption index”.

Recall, that here parameter $\gamma_i > 0$ is the region’s ‘*degree of populism*’, compared to federal authority’s populism. For measuring γ_i we should tell, how the region would distribute an occasional extra rouble between enhancing public goods and reducing tax burden, and compare this ratio with federal legislator’s ratio. Say, if federal authority would spend it 1:1, while region i gives 2/3 to public goods and 1/3 to business, then its populism is $\gamma_i = \frac{2}{1} : \frac{1}{1} = 2$.

May be, such variables can be measured econometrically. Another way is expert estimation of γ_i through interviews. Say, federal legislator should tell us, does he/she suppose regions excessively or insufficiently populist (is $\gamma > 1$?), and to what extent. Vagueness of such expert estimates do not undermine our logic, but rather questions the possibility of making *any* legislative decision on theoretic grounds (anyway, one *must* know this ratio before the constitutional choice).

Another parameter of L_i to be estimated is $\dot{g}_{iT}^* \geq 0$ which is *the share of each additional rouble of region’s budget that goes to public goods*, in other words, it is the marginal share of regional budget spent on household’s needs (not wasted for “luxury”). When discussing γ_i we were dividing an extra rouble between households and businessmen. Now to estimate \dot{g}_{iT}^* we discuss its division between households and corrupted governor, otherwise the idea remains the same. Similar is Edwards and Keen’s “corruption” index \dot{C}_i which is connected with ours in the simple form: $\dot{g}_{iT}^* = 1 - \dot{C}_i$. This \dot{C}_i means “marginal government’s propensity to waste tax revenue”. Like E&K, we hope, that experts in corruption studies can evaluate somehow this ratio. The only difference of our index L_i from E&K’s index \dot{C}_i of corrupt behavior is that we allow for $\gamma_i \neq 1$ populism parameter.

For intuition of how \dot{g}_{iT}^* and γ_i interfere, let us observe polar cases of index L_i . Obviously, L_i becomes zero when the region is completely benevolent: $(\gamma_i, \lambda_i) = (1, 0) \Rightarrow \dot{g}^* = 1$. So, some **tax stimulation is definitely needed under benevolent governments**, when there is some rivalry: $0 < R_i(\cdot)$. This specific result is the essence of typical Oatsian arguments against competition, since, unlike our study, they do assume benevolence and ignore corruption.

The opposite situation, $L_i = 1$ happens when the region is totally corrupted ($\lambda_i = \infty \Rightarrow \dot{g}_{iT}^* = 0$) or totally populist ($\gamma_i = \infty$). In this polar case, and in intermediate cases, policy implications depend upon the right-hand side of our inequality.

2) Now let us **interpret the right-hand side** of our criterion, $R_i(\cdot)$, which is “rivalry index”.

It shows *how seriously all regions do compete* for i -th region’s tax base. To explain it intuitively, suppose that we live in the first region: $i = 1$. Then we can express our rivalry index R_1 related to increase in our tax t_1 as

$$R_1 = \frac{\sum_{j \neq 1} (\dot{V}_{j1} + \dot{u} \dot{g}_j \dot{T}_{j1})}{|\dot{V}_{11}|} \approx \frac{-\dot{V}_{1move} + \sum_{j \neq 1} \dot{u} \dot{g}_j \dot{T}_{j1}}{-\dot{V}_{1close} - \dot{V}_{1stay} - \dot{V}_{1move}}.$$

To comprehend these relations, note that here the numerator is our rivals’ marginal *gains* $\dot{U}_{ji}^{ben}(\bar{t})$ from the increase in our tax. The denominator is the marginal entrepreneurs’ *losses* $|\dot{V}_{ii}(\bar{t})|$ in our region from this extra tax. They are splited into three parts: 1) nation-wide losses $|\dot{V}_{1close}|$, resulting from *ceasing* business that becomes unprofitable; 2) some transfer of welfare of enterprises *remaining* here into our regional tax revenue $|\dot{V}_{1stay}|$; 3) our local (intra-regional) losses of profit of enterprises that *escape* new taxation by moving to other regions $|\dot{V}_{1move}|$. These 3 types of business reaction to the tax rise we should be able to evaluate for estimating index R . The first summand relates to immobile business, idiosyncratic to our region and bringing almost zero net profit. The second one relates to profitable business, not so much mobile as to react to this small tax increase. And the third one relates to very mobile (in current situation) business, that feels almost no difference between staying with us or going elsewhere, since being able to make almost the same net profit here and there. That’s why we have substituted $\sum_{j \neq 1} \dot{V}_{j1} = -\dot{V}_{1move}$; i.e., this part of our lost net profit is a good estimate of our rival’s gains of *entrepreneurial* welfare.

Similarly we should estimate outside *households’* gains $\sum_{j \neq 1} \dot{u} \dot{g}_j \dot{T}_j$ from our tax shift. It is convenient to divide everything by $(\partial T_1 / \partial t_1)$, it gives

$$R_1 = \frac{\partial V_{1move} / \partial T_1}{-\partial V_1 / \partial T_1} + \frac{\sum_{j \neq 1} \dot{u} \dot{g}_j * (\partial T_j / \partial T_1)}{MEB_1 + 1}.$$

Here $MEB_1 := \partial L_{oss1} / \partial T_1 = -1 - \partial V_1 / \partial T_1$ denotes our regional Marginal Excess Burden related to tax shift, due to moving or closing enterprises (the same logic applies when we speak about decreasing *potential* investments, settling elsewhere or just ceasing, instead of discussing existing enterprises). How can we predict the ratio $(\partial T_j / \partial T_1)$ related to the impact of our tax revenue onto other regions’ revenue? When regions have more or less the *same tax rate*, then it is reasonable to suppose that the same investment projects applied there make almost the same tax revenue as here: $(\sum_{j \neq 1} \partial T_j / \partial T_1) \approx k_{moveT1} := (\partial T_{1move} / \partial T_1)$, otherwise we should multiply k_{moveT1} by the relation of tax rates. To estimate ‘budget hunger’ variable $\sum_{j \neq 1} \dot{u} \dot{g}_j = \dot{u} \dot{g}_2$ use equilibrium conditions $\dot{u}_j(\bar{g}_j) = -\dot{V}_{jj}(\bar{t}) / (\gamma_j \dot{T}_{jj}(\bar{t})) = (MEB_j + 1) / \gamma_j$. Additionally suppose for simplicity that there is only one outside region (No 2), or all of them are similar to No 2, then

$$R_1 \approx \frac{\partial V_{1move} / \partial T_1}{-\partial V_1 / \partial T_1} + \frac{k_{moveT} \dot{g}_2 * (MEB_2 + 1) / \gamma_2}{1 + MEB_1} \approx k_{moveV1} + k_{moveT1} \frac{\dot{g}_2}{\gamma_2} * \frac{1 + MEB_2}{1 + MEB_1}.$$

Here parameter k_{moveV} can be expressed and evaluated in terms of changes in regional net profit after some tax shift as $k_{moveV1} \approx (\Delta V_{1move})/(\Delta V_{1all}) = (V_{1mov}^{new} - V_{1move})/(V_{1all}^{new} - V_{1all})$, or as the same proportion in percentage terms. Similarly $k_{moveT1} \approx (\Delta T_{1move})/(\Delta T_1)$. (There is an example in Appendix 3 additionally illustrating calculations of this index R , and showing that sometimes $\Delta T_{1move} > \Delta T_1$).

Summarizing, to measure rivalry we should be able to evaluate 1) benevolence (\dot{g}_2/γ_2); 2) marginal excess burden, and its splitting into mobile and immobile parts; 3) loss of net profit in the region due to additional tax and its splitting into mobile and immobile parts. Under simplifying assumptions it is sufficient to know mobility of net-profit $k_{moveV}(t)$ and mobility of taxes $k_{moveT}(t)$ (both may depend upon current situation t), plus benevolence.

One can compare the above formula with the index of rivalry from E&K: $\frac{MEB}{(1+MEB)}$. Though there is similarity, but the notion MEB have different meaning in their general-equilibrium context, as well as capital mobility. Besides, the difference arise because E&K evaluate multilateral tax increase, and do not allow for populism.

This discussion show that rivalry index R increases with tax-mobility ratio; with profit-mobility ratio and with benevolence, becoming *greater than 1* **under sufficiently high mobility** ($k_{moveT} > 1$) **and sufficiently benevolent governments**, in this case $L < R$. **Then small tax increase is definitely welfare- justified in tax-competition situation.**

Thus we have repeated typical Oatsian result, expanding it at the same time to a vast range of situations with rather high mobility and with reasonable corruption and populism.

On the contrary, rivalry index decreases with non-benevolence (γ/\dot{g}) and with immobility, that discourages tax stimulation.

5 Co-ordination, welfare comparisons ‘in large’, and heterogeneity

Now we turn to the main topic: using our ‘in-small’ welfare diagnosis for the global comparison of federalism regimes.

5.1 What is co-ordination equilibrium?

What should be the realistic concept for “co-ordination” of regions? Should we assume a *cooperative* solution, or *non-cooperative* one? We suppose the latter being more realistic to consider. At least, for a large group (say, $m > 3$) a theorist *should* mention some non-cooperative procedure converging to a cooperative solution suggested. What is the procedure in reality?

We do not know exactly. There are some formal and informal political procedures. Formally, in Russia regions make their propositions for the budgetary and legislative issues, and *vote* for the budget, or for a bill on taxes, in the upper chamber of the parliament. Less formally, any bill goes through parliamentary conciliatory committees and governmental corridors, through ‘log-rolling’ and lobbying.

To model this game, we suppose some *voting* procedure to be a good proxy. Let us be more specific. First, we doubt that sufficiently many regions (e.g., there are 89 regions in Russia) can reach an agreement about *individualized* tax rates for all regions. Rather they

can discuss *uniform* tax rate, one for all regions. Second, what kind of voting is a good proxy for the real political battle? Sometimes it is realistic to suppose *majority rule* (it amounts to median-vote under our assumptions, assuring single-peakedness of preferences for a tax rate). Instead we can suppose somewhat more general and realistic: ‘shifted-median’ or α -voting procedure. Of course it is imaginary, being only a proxy for uneven bargaining power of regions.

Definition 2 Procedure of shifted-median voting (or α -voting) means that all suggestions (bids) t_i of agents about scalar variable τ are ordered as $t_1 \leq t_2 \leq \dots \leq t_m$ and normalized on the $[0, 1]$ interval as: $t_1 \rightarrow 0$, $t_2 \rightarrow 1/(m-1)$, $t_3 \rightarrow 2/(m-1)$, ..., $t_m \rightarrow 1$. Then the suggestion closest to the apriori given parameter $\alpha \in [0, 1]$ wins (in the case of the two closest bids, one of them wins randomly).²⁰

In particular, $\alpha = 1/2$ is the usual median-vote rule, reflecting equal rights or powers. Another sample is when regions preferring smaller tax-rates (often it is rich regions) have more bargaining power than others, that can be described by $\alpha < 1/2$. (At least, rich regions are powerful in Euro-Union and in Russia.)

The third important question of political mechanism is - how proceeds of taxes are (re)distributed? We shall suppose here *constant* redistribution transfers, fixed once for all times (that is equivalent to *no redistribution* in respect of stimuli). It obviously contradicts the Russian reality, where we see large inter-budgetary transfers and subsidies changing in time. However, we can not say anything distinct about this transfer-game, and restrict our attention here on the game without fight for transfers (hopefully approximating the real game). Under these assumptions we can formulate

Definition 3 Shifted-median voting rule α and subsidy values s_i given, *voting-co-ordination solution* is the triple $(\bar{\tau}, \bar{g}, \bar{l}) \in R_+^{2m+1}$ such that for each i , for given $t_i = \bar{\tau} \forall i$, vector (\bar{g}_i, \bar{l}_i) solves the ‘spending problem’:

$$\begin{aligned} \gamma_i u_i(g_i) + \lambda_i w_i(l_i) &\rightarrow \max_{(g_i, l_i)} s.t. \\ g_i + l_i &= T_i(t_i, t_i, \dots, t_i) + s_i, \quad g_i \geq 0, l_i \geq 0. \end{aligned}$$

At the same time, scalar $\bar{\tau}$ is the α -vote out of m votes (t_1, \dots, t_m) , each t_i being the solution to the ‘uniform tax-rate-bid problem’:

$$U_i = \beta_i V_i(t_i) + \gamma_i u_i(g_i^*(t_i, \gamma_i, \lambda_i)) + \lambda_i w_i(T_i(t_i, t_i, \dots, t_i) + s_i - g_i^*(t_i, \gamma_i, \lambda_i)) \rightarrow \max_{t_i \in [0, 1]},$$

where $(g_i^*(t_i, \gamma_i, \lambda_i), l_i^*(t_i, \gamma_i, \lambda_i))$ denotes the solution to the above spending problem (it is the response-function).

We must note that here any α -voting co-ordination solution exists and describes in fact a dominant-strategy equilibrium, not only Nash. Indeed, under our assumptions the profile of preferences is single-peaked w.r.t. tax rate. Therefore, by the same logic as under median voting, Condorset paradox is excluded, the equilibrium exists, and sincere voting is a dominant strategy.

²⁰One can see that this procedure is not exactly the same as d -majority rule (see e.g. Enlow, 1997), and that median-voting game can differ from majority voting in the absence of single-peakedness.

5.2 Externalities at work: does co-ordination really increase taxes?

Recall that our logic of constitutional choice requires to find which of the two compared states of economy has higher average tax rate (in some sense). This paragraph, in essence, performs almost-standard exercise in externality analysis, becoming not so evident under our assumptions. The general idea is usual in ‘tax-competition’ analysis. Namely, when choosing a tax rate, each region ignores positive externality on other regions, i.e., capital flight from this region to its partners, induced by the tax. Therefore, common wisdom is that there should be smaller taxes under competition, than under co-ordinated decision of regions. However, let us see that in voting game this general tendency works not so easily, or only under specific hypotheses. In some cases common wisdom can be wrong because our (voting) ‘co-ordination solution’, unlike usual one, is a non-cooperative one, that matters in heterogeneous country, as we shall see now.

Let us show that regional heterogeneity itself may make average tax rate at co-ordination *smaller* than at competition. The idea is simple, and works even for $\alpha = 1/2$, that is median-voting rule. When externality effect is small or absent, then it is the relation between median tax and average tax (among bids of all regions), that determines, whether average taxes increase or decrease after fiscal regime switches from competition to co-ordination. Say, among three players there can be one region choosing very high tax in competition and two regions choosing very small tax rates. After switching to co-ordination under absent externalities, the ‘small’ regions just force the only high-tax region to follow the majority preferences, and average tax rate decreases (even the whole vector of tax rates non-strictly decreases!). Weighted average tax-rate also decreases, when non-benevolence of regions has the same sign, for example all three shows over-taxing ‘in-small’. Similarly, some externality can be outweighed by an α -voting rule or other non-cooperative solution in sufficiently heterogeneous country.

So, common wisdom about higher taxes at co-ordination is not guaranteed in our heterogeneous setting. Are there any general conditions on voting mechanism and on other parameters sufficient for such ‘natural’ outcome? Now we only can formulate the two rather special cases.

5.2.1 One case of tax rise: (almost) symmetric equilibria

One possible situation when co-ordination increase taxes, is some sort of symmetry of equilibria. It definitely happens not only when regions are identical (like in E&K and in many other papers), but also in a broader class of cases. Indeed, some regions can be *different* in many respects, but have *similar* taxes. It seems realistic, first of all, because competition imposes strong symmetrizing forces. Besides, on other grounds, one region may have larger populism, while the other have larger luxury-loving, resulting in the same ‘budget hunger’ and the same tax rate. Anyway, this assumption is much more plausible than uniformity of all parameters, being as much useful, to make the common-wisdom prediction that we deserve: more taxes at co-ordination.

At first glance the comparison is trivial. Like in textbook “cartel instability” story, all agents must decrease prices (taxes) when becoming free of cooperative discipline. However, our co-ordination solution is determined by the median voter or the α -voter only. Yes, she tends to decrease tax rate after becoming free. But nothing particular can be said about her partners, who did not participate in choosing the co-ordination point! Under

high heterogeneity among regions it may happen that some regions increase taxes, and what happens next, how it influences all others?

So, we need symmetry assumption and a mild additional assumption (Assumption 4) on externality effect to escape ambiguity.

Now we start proving the named “common wisdom” with characterizing (symmetric) co-ordination solution. Let us study i -th region’s objective function, when it chooses its vote $\theta = \theta_i \geq 0$ for tax level, and supposes itself to be a median voter or α -voter.

Take a region’s α -voting problem:

$$U_i = V_i(\theta, \dots, \theta) + \gamma_i u_i(g_i^*(T_i^*(\theta, \dots, \theta)) + \lambda_i w_i(T_i^*(\theta, \dots, \theta) - g_i^*(T_i^*(\theta, \dots, \theta))) \rightarrow \max_{\theta}$$

Derive the co-ordination F.O.C. w.r.t. common tax rate θ :²¹ $\frac{\partial}{\partial \theta}[V_i(\theta, \dots, \theta) + \gamma_i u_i(g_i^*(T_i^*(\theta, \dots, \theta)) + \lambda_i w_i(T_i^*(\theta, \dots, \theta) - g_i^*(T_i^*(\theta, \dots, \theta)))]$

$= 0$. Use F.O.C. of spending problem to eliminate w_i and obtain $\sum_j \dot{V}_{ij}(\theta, \dots, \theta) + \gamma_i \dot{u}_i \sum_j \dot{T}_{ij}^*(\theta, \dots, \theta) = 0$. Reformulating this for i -th summand we get

$$\dot{V}_{ii}(\theta, \dots, \theta) + \gamma_i \dot{u}_i \dot{T}_{ii}^*(\theta, \dots, \theta) = -[\sum_{j \neq i} \dot{V}_{ij}(\theta, \dots, \theta) + \gamma_i \dot{u}_i \sum_{j \neq i} \dot{T}_{ij}^*(\theta, \dots, \theta)].$$

We compare this to F.O.C. of competition:

$$\dot{V}_{ii}(\tau, \dots, \tau) + \gamma_i \dot{u}_i \dot{T}_{ii}^*(\tau, \dots, \tau) = 0.$$

To compare solutions θ and τ to these two equations, we are interested in monotonicity of the left-hand side looked upon as a function ϕ :

$$\phi(\tau) := \dot{V}_{ii}(\tau, \dots, \tau) + \gamma_i \dot{u}_i (g_i^*(T_i^*(\tau, \dots, \tau)) \dot{T}_{ii}^*(\tau, \dots, \tau)).$$

(We can not get monotonicity directly from concavity of region’s objective function used in competition, since here we must include externalities. Similarly, we can not directly use co-ordination’s second-order conditions.) We take derivative:

$$\phi'(\tau) = \sum_k \ddot{V}_{iik} + \gamma_i \ddot{u}_i \dot{g}_i(\sum_k \dot{T}_{ik}^*) \dot{T}_{ii}^* + \gamma_i \dot{u}_i (\sum_k \ddot{T}_{iik}^*) = (\ddot{V}_{iii} + \gamma_i \ddot{u}_i \dot{g}_i(T_{ii}^*)^2 + \gamma_i \dot{u}_i \ddot{T}_{iii}^*) + \sum_{k \neq i} \ddot{V}_{iik} + \gamma_i \ddot{u}_i \dot{g}_i(\sum_{k \neq i} \dot{T}_{ik}^*) \dot{T}_{ii}^* + \gamma_i \dot{u}_i (\sum_{k \neq i} \ddot{T}_{iik}^*)$$

The first summand here is the second derivative of competition problem, it must be negative by concavity. The rest consist of cross-derivatives related to externalities. Economic considerations enable to restrict their values with **Assumption 4** (negative \ddot{V}_{iik}). Indeed, negativity of $\ddot{V}_{iik} = \ddot{V}_{iki}$ means that when I am rising my tax, my marginal revenue from my rival tax-rise decreases. It sounds plausible: the greater my tax - the less attractive is my region. With similar reasons for \ddot{T}_{iik}^* , we use **Assumption 4** on cross-derivatives, and former assumptions on $\ddot{u}_i, \dot{T}_{ij}^*$, to get $\phi'(\tau) < 0$, i.e., decreasing $\phi(\tau)$. Using now $\sum_{j \neq i} \dot{V}_{ij}(\theta, \dots, \theta) + \gamma_i \dot{u}_i \sum_{j \neq i} \dot{T}_{ij}(\theta, \dots, \theta) > 0$, we obtain finally the needed estimate $\tau < \theta$.

Now, to slightly generalize our result, recall that we have assumed twice continuously differentiable functions. So, we must believe in continuity of equilibria w.r.t. parameters. This enables to say ‘almost’ in the following assertion obtained.

(Almost) symmetric tax-competition equilibrium

$(t_1, \dots, t_m) : t_i \approx \sum_j t_j / m = \tau$ **can not have higher taxes than voting-co-ordination solution θ , i.e., $\tau \leq \theta$, and in the case of non-zero externalities it has strictly lower average tax rate: $\tau < \theta$.**

²¹Here and everywhere, to bravely use F.O.C. we must be sure that the solution is not a border one, i.e., taxes are positive. This follows from Inada’s condition of Assumption 1 when all transfers $s_i = 0$. In more general case it is an additional assumption used throughout.

Note once again, that we do not assume identical or similar regions here, only taxes are assumed similar, that's why it is a sufficiently broad generalization of "common wisdom".

5.2.2 Another special case: race to the bottom

Together with symmetry, there is one more situation enabling clear comparison of competition and co-ordination in tax rates. It is the case of "race to the bottom", i.e., severe competition yielding (almost) zero taxes at equilibrium. What characteristics of tax-base function provide such outcome?

It is interesting that neither "rivalry" of regions at equilibrium point, nor "mobility ratio" index (explained above) take responsibility for this effect. Indeed, both these characteristics relate to *first* derivatives of V, T , while Bertranian effects are due to very high *second* derivative of revenue V , i.e., the first derivative of demand. So, to describe conditions for "race to the bottom", we must somehow characterize this 'temp' of tax base mobility. What makes competition really cut-throating?

Most evident example is the conventional "Bertrand's competition" hypothesis.²² It says that when a tax t_i is higher than some other tax: $t_i > \min_j \{t_j\}$, then the related welfare and tax base become zero: $V_i(t) = 0, T_i(t) = 0$, while both functions take positive values when all regional taxes coincide [$t_1 = t_2 = \dots = t_m \Rightarrow V_i(t) > 0, T_i(t) > 0 \forall i$]. Besides, there exists $\varepsilon > 0$, such that any (even infinitesimally small) tax reduction, started by a region in profitable situation, brings it finite ε -increase of both functions. I.e. $T_i(t) > 0 \Rightarrow [T_i(t_1, \dots, t_i - \delta, \dots) > T_i(t) + \varepsilon \forall \delta > 0]$. Obviously, this hypothesis contradicts continuity and convexity of net-profit and tax-revenue functions, but we ignore this shortcoming here, since equilibria existence follows from Bertranian hypothesis otherwise.

Under Bertranian hypothesis equilibrium exists and has zero taxes (it can be proved standardly).

By the way, such equilibrium may be good for welfare, in contrast to common wisdom. There is no clear arguments why zero-tax competition point must lie on lower indifference curve than co-ordination point. We do not suppose the tax discussed here to be *the only* budgetary source for our regions. There can be transfers and other sources. So it is not clear why our households must necessarily milk *this* tax base for their needs. 'Race to the bottom' may be good or bad, depending upon households' and legislator's preferences and possibilities.

Only when at co-ordination equilibrium we had positive taxes and under-taxing diagnosis, our logic enables to look on race to the bottom as definitely harmful.

6 Conclusions

We have followed the idea of Edwards and Keen, to look on federalist dilemma between tax competition and tax co-ordination like on quantitative question: we should *measure* some parameters of economy to decide, what is better for particular country. Our paper

²²A weaker, much more reasonable hypothesis of the same kind studied in the Appendix is "great demand elasticity" ("severe competition"). It keeps demand continuity, being formulated in terms of infinitely high derivatives, but also contradicts our assumptions on continuous derivatives and concavity, maintained in other sections (so, it may also challenge equilibria existence). Quasi-Bertraian competition occurring under this assumption turns out non-trivial: 'race to the bottom' is not guaranteed.

attempts to make this measurement more practical, and extends this approach in several aspects, important for federation like Russia: 1) regions may be supposed non-identical (*heterogeneous country*); 2) taxation of *particular industries* or economy sectors can be studied as well as taxation of the whole economy; 3) in addition to corruption, another big source of non-benevolence - *populism* - was included into analysis; 4) *big jump* from competition to co-ordination is discussed in addition to small or “gradual” regulation. For this task we construct an appropriate model, which is a partial equilibrium model instead of traditional general-equilibrium one.

The dilemma is studied in two stages. First, we decide, whether we observe or not local (marginal) *over-taxing* at competition equilibrium. For this we must measure and compare two local economic indices: “interregional rivalry” and “non-benevolence” (a method is suggested to estimate them). If “rivalry” index is smaller than “non-benevolence” index, then it is the sign of over-taxing in our economy, and taxation should be somewhat reduced, because their difference measure marginal social welfare from additional taxation. If this difference is in average negative among regions, and the weighed-average tax rate at competition is less than tax rate at would-be co-ordination equilibrium, then “separation formula” shows that there is no need in switching from competition to co-ordination.

Philosophically, the model shows how corruption, tax-exporting motives, and populism can outweigh capital-flight impact on public welfare, and when fiscal federalism and its specific form may be **good or bad for certain country or economy sector**.

It is shown what should we measure to make a diagnose.

7 Appendices

7.1 Appendix 1: deriving capital demand from investment projects

Let us explain how “regional demand for capital” and its “mobility” follow from initial data on investment projects, and why our assumptions on demand function D_i can be justified.

Regional investment projects comprise technologies for making profit and create demand for capital. Treating “project” as economic agent (entrepreneur), we have in mind somebody knowing the technology and eager to implement it with borrowed capital, or with his/her own capital. Typical project is characterized by fixed total amount of required investment and by its expected *gross* profitability which we define as revenue net of material costs, federal taxes and wages (it is unusual to exclude federal taxes from “gross profit” but it is convenient here). Inter-regional variations of profitability express a project’s *mobility*. A project may yield the same gross profit in different regions (perfect mobility), or different profits (limited mobility), or no profit in all but one region (absent mobility, idiosyncratic project).

The table below with regions $i = 1, 2, 3$ and investment projects $j = 1, 2, 3, 4$ illustrates this idea on imaginary sample. Complete immobility would result in diagonal matrix of gross profitability $[\pi_{ji}]$ (profit of j -th project in i -th region), while perfect mobility can be reflected by a matrix with all columns $[\pi_i]$ coinciding. Only the 2-nd project is perfectly mobile here, and only the 4-th project is immobile.

For analyzing the reaction of investment to taxes we need a compact exposition of the projects data, i.e., the region’s demand-for-capital function $D_i(p_0, t_1, \dots, t_m)$, which is the “response-function” of all the agents (projects) to interest rate p_0 and to taxes t_i . To con-

Prj No	Invest- ment	Expected gross/additional profitability (for $p_0 = 0$)		
		Region 1 ($t_1 = 0.00$)	Reg 2 ($t_2 = 0.01$)	Reg 3 ($t_3 = 0.05$)
1	$d_1 = 10$	$\pi_{11} = 0.40, a_{11} = -0.01$	$\pi_{12} = 0.42, a_{12} = +0.01$	$\pi_{13} = 0.30, -0.15$
2	$d_2 = 20$	$\pi_{21} = 0.15, a_{21} = +0.01$	$\pi_{22} = 0.15, a_{22} = -0.01$	$\pi_{23} = 0.15, -0.06$
3	$d_3 = 30$	$\pi_{31} = 0.20, a_{31} = +0.09$	$\pi_{32} = 0.12, a_{32} = -0.09$	$\pi_{33} = 0.13, -0.02$
4	$d_4 = 40$	$\pi_{41} = 0.11, a_{41} = +0.11$	$\pi_{42} = 0.00, a_{42} = -0.12$	$\pi_{43} = 0.00, -0.05$

Table 1: Example of investment projects region-specific profitability.

struct this demand function from the projects data we suppose that the goal of an agent named “investment project” is the *net* entrepreneurial residual (“net profitability”), which is gross profitability net of taxes and interest rate (treated as fixed). For the case of property tax (tax proportional to capital, or “unit tax”) we can express this profitability as $p_{ji}(t) := \pi_{ji} - t_i - p_0$, while for least distortionary *ad valorem* taxation net profitability is $p_{ji}(t) := (1 - t_i)(\pi_{ji} - p_0)$.²³ Any project chooses the best region, i.e., a region $m(j, t)$ which demonstrates maximal net profitability $p_j^*(t) := \max_i p_{ji}(t)$ for this project, or, in other words, maximal “additional profit” a_{ji} , relatively to the best other region in current situation $a_{ji}(t) := p_{ji} - \max_{s \neq i} p_{js}(t)$. If $p_j^* \geq p_0$ then the project can be fulfilled, and it demands as much as d_j investment from the capital market. To get regional capital demand we just summarize these demands for all profitable projects choosing this region:²⁴

$$D_i(p_0, t) = \sum_{j: \{p_j^*(t) \geq p_0 \text{ \& } i=m(j,t)\}} d_j .$$

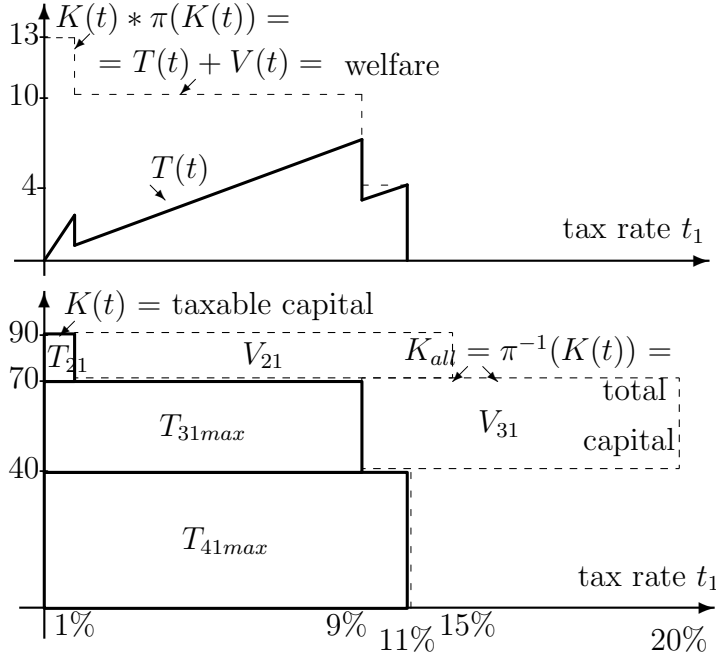
In other words, i -th capital demand or “investment curve” can be built (for fixed other taxes) similarly to demand curve of any market where each consumer takes one unit. First, order the “willingness to pay” or positive additional profitabilities of projects settled in our region under zero our tax $t_i = 0$: $a_{j_1 i}(0, t_{-i}) \geq a_{j_2 i}(0, t_{-i}) \geq \dots \geq 0$. Second, represent each project by a rectangle $a_{ji}(0, t_{-i}) \times d_j$ and stack these rectangles one onto another starting from the most additionally-profitable at the bottom to the least profitable at the top. For the the 1-st region of Table 1(7.1) it gives the curve of “taxable capital” $K_{1taxable}(t_1) := D_1(p_0 = 0, t_1 = 0, t_2 = 0.01, t_3 = 0.05)$ exhibited on Fig. 2 by solid line. Its inverse function is $a_1(k)$ where $k = 4, 3, 2$ are names of the projects placed in proper order on ordinate. The area of rectangles describe maximal-possible tax revenue T_{j1} from each project. In contrast, “welfare of capital” mapping $K_{all}(t)$ (dash line) we get when exhibiting *gross* profitability $\pi_j(k)$ of all these projects on the abscissa, and area V_{21}, V_{31} , describe non-taxable welfare of entrepreneurs, and their *mobility*. The upper graph of this figure contains the integrals of the curves from the lower graph, to describe tax revenue $T_1(t)$ and entrepreneur’s residual $V_1(t)$ in our region 1. For sufficiently many projects such capital curves should become smooth as on Fig. 3).

Note that different taxation schemes (unit taxes like property tax, or *ad valorem* taxes like profit tax) generate *different* demand functions D from the *same* projects’ information. However, in any case the demand function should satisfy our assumptions: under any taxes demand D_i is bounded from above, decreasing w.r.t. own tax t_i , and increasing w.r.t. other

²³By the way, our model can be applied to the case when one region use property tax, while another uses profit or any other tax. Only our statements about “coinciding” taxes will lose meaning.

²⁴Here we suppose that maximum $m(j)$ is unique, but the method can be generalized.

Figure 2: One region's capital demand and welfare generated by 3 projects of Table 1.



taxes. Besides, the amount of projects (or profit) $\sum_{j \neq 1} \Delta D_j(p_0, t)$ moving *to* other regions $j \neq 1$ after the increase in our tax Δt_1 is not greater than the amount of projects $|\Delta D_1(p_0, t)|$ gone away *from* our region 1.

7.2 Appendix 2: deriving welfare from capital demand

Tax revenue $T_i(t_i)$ and consumer surplus (entrepreneurial welfare $V_i(t_i)$) can be derived almost standardly from demand. However, when national-wide interest rate $p_0 = p_0(t)$ is not a constant w.r.t. all taxes (such variability is assumed in most tax-competition studies), these notions need calculating capital-market equilibrium that becomes therefore a detail of tax-competition equilibrium. In this case we should assume some capital-supply function $S_{upp}(p_0)$, non decreasing w.r.t. interest rate p_0 . Standardly, competitive interest rate function $p_0^*(t)$ defined by equilibrium on capital market under taxes t is characterized by equation:

$$p_0^* : S_{upp}(p_0^*) = \sum_{i=1}^m D_i(p_0^*, t_1, \dots, t_m).$$

This competitive-interest-rate function $p_0^* = p_0^*(t)$, is well-defined and single-valued when capital equilibrium exists and happens unique (that we shall assume). Therefore, assuming intelligent enough governors to think of equilibria, we can substitute this market-reaction function into the capital-demand function, and introduce the investment function K depending only upon taxes as compound function

$$K_i(t_1, \dots, t_m) := D_i(p_0^*(t), t_1, \dots, t_m).$$

We often use simplified notation $K_i := D_i$, dropping argument $p_0^*(t)$ after this explanation, but we should explain why we eliminate p_0^* from calculating welfare. We everywhere

exclude dividends of capital owners from calculations of regional welfare, performed by the governors and by the “benevolent legislator”. One reason is that large part of investors can be outsiders. Another reason is that most realistic is to consider the economy in question “open”. It means that investors have enough opportunities to apply capital outside our “economy” in other sectors, abroad or in consumption. Then capital supply is very elastic and $p_0^*(t)$ appears almost constant, so any taxes that we apply make no difference for investors’ welfare (but for the case of insider investors and small elasticity of $S_{upp}(\cdot)$ our calculation of welfare has some bias).

To derive welfare from capital, consider first the simpler case: property tax, that means that tax-base is the amount of applied capital itself and tax revenue is defined as $T_i(t) := t_i D_i(p_0^*(t), t) = t_i K_i(t)$.²⁵

Entrepreneurs’ monetary welfare remains of profit after taxation and paying interest (which plays the role of costs), it is expressed like “consumer surplus” as in Appendix 1:

$$V_i(t) = \int_0^{K_i(t)} \pi_i(\kappa) d\kappa - t_i K_i(t),$$

where κ takes values from the most additionally-profitable unit of capital to the least one, and $\pi(\cdot)$ denotes gross profit as a function of these values (see our figures and Appendix 1).

Similarly we can derive tax revenue and welfare also in the case of “least-distortive” schemes of taxation. The least distortive method is taxing profit net of interest.²⁶ It gives $T_i(t) := t_i \int_0^{K_i(t)} \pi_i(\kappa) d\kappa$, $V_i(t) := (1 - t_i) \int_0^{K_i(t)} \pi_i(\kappa) d\kappa$. (Caution: as explained above, capital function $K_i(t)$ *differ* here from $K_i(t)$ of distortionary taxation, though investment projects are the same.)

7.3 Appendix 3: Example of rivalry index calculation

Example. Figure 3 illustrates capital demand, welfare and rivalry index for the case when some region (regional index $i = 1$ is dropped on the picture and our rivals’ taxes are supposed fixed) has 3 times more mobile enterprises than immobile, within each part of taxable demand (thick solid curve K_{taxmob}), from the most profitable to the least profitable ones, so proportion $k_{mob/imm} = 3/1$ everywhere. The immobile capital is presented by the solid curve K_{immob} on the *lower* plot. We assume also that proportion in taxable/nontaxable profit of mobile enterprises is everywhere 1/1 (for each unit), that can be seen from mobile profit (dashed) curve K_{allmob} being 2 times righter than the taxable curve K_{taxmob} (see Appendix 1 for explanation). We assume taxation in the form of property-tax: $T(t) = tK(t)$. It means that region’s tax revenue $T = T_{mobile} + T_{immob}$ (two solid rectangulars) is proportional to the tax rate t and to capital applied K .

²⁵ Capital-income tax also may work similarly to property tax discussed here. In this case our variable t_i should not be understood as the tax itself, the model application should be adjusted.

²⁶ The same happens if we were able to apply discriminating lump-sum taxes, leaving each investment project (see Appendix 1) with positive net entrepreneur’s revenue (with profit more than normal profit). Then the country’s total tax base (total capital applied) could be independent of taxes! Lafferian tradeoff disappears under taxation of rent. This scheme of taxation is absolutely “non-distortive” *only* when all taxes are the same. Indeed, the same project can bring more profit in one region, but may be forced by high tax to move to another one, inducing welfare losses.

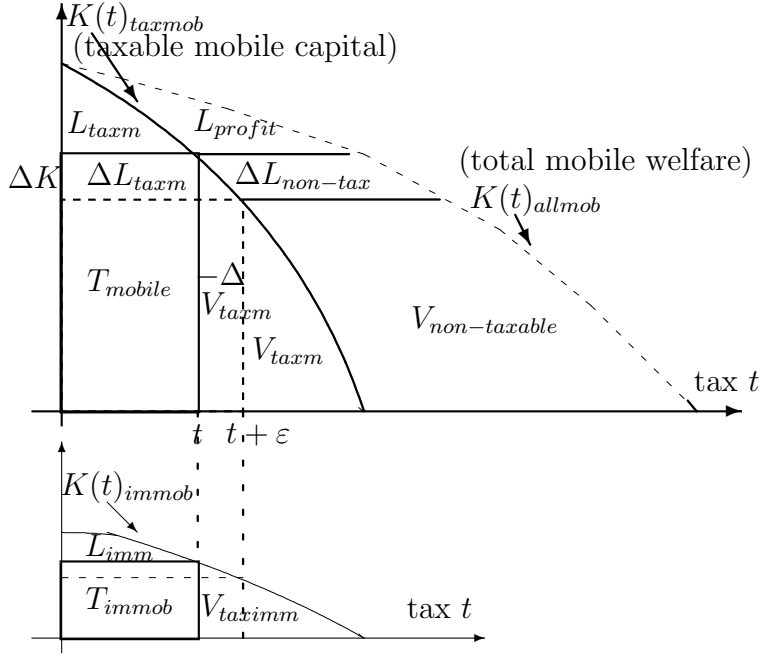


Figure 3: Taxable and non-taxable, mobile and immobile demand for capital, and related welfare indicators T, V for one region.

Now, let us realise what happens with welfare after a tax rise. Suppose that tax rate in our region has risen from t to $t + \varepsilon$, as on the picture, and the related reduction in mobile capital amounted to $\Delta K = -1$ (so, immobile $\Delta K_{imm} = -1/3$). Consequently some part $|\Delta V_{taxm} + \Delta V_{imm}|$ of taxable profit (mobile and immobile) is transferred from entrepreneurs' welfare into additional tax revenue. Yet, some tax revenue $\Delta L_{taxm} + \Delta L_{imm}$ together with some profit ΔL_{prof} is transferred into our additional deadweight-loss, partially $(\Delta L_{taxm} + \Delta L_{prof})$ benefitting to other regions. We shall assume these quantities to be $\Delta L_{taxm} = 2 = \Delta L_{non-tax}$, $\Delta V_{taxm} = -3$, so $\Delta L_{imm} = \Delta L_{taxm}/k_{mob/imm} = 2/3$, $\Delta V_{imm} = -1$. Then additional tax revenue is $\Delta T = \varepsilon \dot{T} \approx |\Delta V_{taxm}| - \Delta L_{taxm} + |\Delta V_{imm}| - \Delta L_{imm} = 4/3$. At the same time, our additional deadweight-loss contains one more term $\Delta L_{non-tax} = |\Delta V_{non-tax}| = |\Delta V_{move}| = 2$ connected with mobile profit that moves to other regions. It is not a loss for the whole country, as well as $\Delta L_{taxm} = \Delta T_2 = 2$, in contrast with ΔL_{imm} .

To illustrate the rivalry index resulting from our tax increase, we calculate $\Delta V = \varepsilon \dot{V} \approx -\Delta V_{taxm} - \Delta V_{imm} - \Delta L_{non-tax} = -3 - 1 - 2 = -6$, $\Delta L = \Delta L_{taxm} + \Delta L_{non-tax} + \Delta L_{imm} = 2 + 2 + 2/3 = 14/3$, $MEB = \Delta L/\Delta T = 14/4$. Further, $k_{moveT} = \Delta L_{taxm}/\Delta T = 3/2$, $k_{moveV} = |\Delta L_{non-tax}/\Delta V| = 1/3$. Then, assuming our rival region No 2 having tax rate 2 times more than our's, rivalry index can be expressed as

$$R_1 = \frac{\Delta L_{non-tax}/\Delta T + 2 k_{moveT} \dot{u}_2 \dot{g}_2}{1 + MEB_1} = \frac{1}{3} + \frac{2}{3} \dot{u}_2 \dot{g}_2, \text{ or exploiting } MEB_1 = MEB_2 \text{ as}$$

$$R_1 = k_{moveV} + 2 k_{moveT} \frac{\dot{g}_2}{\gamma_2} = \frac{1}{3} + 3 \frac{\dot{g}_2}{\gamma_2}.$$

So, one can calculate R_1 using either our knowledge about 'budget hunger' $\dot{u}_2 \dot{g}_2$ in our rival region No 2, or knowledge about its marginal excess burden, populism and corruption,

in addition to our named economic parameters.||

7.4 Appendix 4: concavity of indirect welfare function

Here we are going to derive the sufficient conditions for concavity of indirect welfare function $U_{all}^*(t) = \sum_i [V_i(t) + u_i(g_i^*(T_i^*(t), \gamma, \lambda))]$. Concavity of V, T , concavity and monotonicity of u was already supposed. Superposition of concave monotone functions enjoys the needed property, so one of possible sufficient conditions for concavity of the whole U_{all}^* is the concavity and monotonicity of $g_i^*(.)$. Recall, that this function is the argmaximum of the region's spending problem:

$$\begin{aligned} \gamma_i u_i(g_i) + \lambda_i w_i(l_i) &\rightarrow \max_{(g_i, l_i)}, \text{ s.t.} \\ g_i + l_i &= T_i(t_i, t_i, \dots, t_i) + s_i, \quad g_i \geq 0, l_i \geq 0. \end{aligned}$$

This problem, in essence, is the standard consumer's problem with separable utility function, composed additively from two "satisfaction functions" u, w , that bore the demand functions $g_i^*(.), l_i^*(.)$ for the two commodities. The demand monotonicity in income T is standard, while $g_i^*(.)$ concavity, as well as concavity of $l_i^*(.)$, depends upon relative characteristics of satisfaction functions" u, w (we drop index i). When both are homothetic of the same degree, then both demands are linear w.r.t. income T . When one of them is "**more quickly satiable**" in some sense, then the demand for this commodity is concave. To see, what this satiability means in terms of derivatives, differentiate the F.O.C., and reformulate to get the first derivative of g in the form $\dot{g}_{iT}^*(T) := \frac{\lambda_i \ddot{w}(l_i)}{\lambda_i \ddot{w}(l_i) + \gamma_i \ddot{u}(g_i)}$. Differentiating this once again we get the necessary and sufficient condition for concavity of $g_i^*(T)$ in the form:

$$\gamma_i \ddot{w}(l_i) \ddot{w}(l_i) \leq \lambda_i \ddot{u}(g_i) \ddot{u}(g_i) \quad \text{or} \quad \frac{\dot{w}(l_i) \ddot{w}(l_i)}{(\ddot{w}(l_i))^2} \leq \frac{\dot{u}(g_i) \ddot{u}(g_i)}{(\ddot{u}(g_i))^2},$$

(that can be monotonically related to the first derivative of the Pratt's measure of functions u, w .)

To see example where the concavity condition holds or not, take functions like $u(x) = x^{1/2}$ and $w(x) = x^{1/3}$. For small $x \in (0, 1]$, our inequality is violated, and \dot{g}_{iT}^* is convex, while for sufficiently high arguments our $\dot{g}_{iT}^*(.)$ becomes concave as we need, and demand with satisfaction function $u(x) = x^{1/2}$, becomes "more quickly satiable" than $w(x) = x^{1/3}$.

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